Exercise 10

Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1.

$$e^x \cos x \approx 1 + x$$

Solution

Plugging in $x = 0$ to the function yields $e^0 \cos 0 = 1$, so $(0, 1)$ is the point on the curve that the tangent line goes through. Taking the derivative of the function yields

$$\frac{d}{dx}(e^x \cos x) = \left[\frac{d}{dx}(e^x)\right] \cos x + e^x \left[\frac{d}{dx}(\cos x)\right] = (e^x) \cos x + e^x(-\sin x) = e^x(\cos x - \sin x).$$

Set $x = 0$ to get the slope of the tangent line.

$$\left.\frac{d}{dx}(e^x \cos x)\right|_{x=0} = e^0(\cos 0 - \sin 0) = 1$$

Use the point-slope formula to get the equation of this line.

$$y - 1 = 1(x - 0)$$

$$y - 1 = x$$

$$y = x + 1$$

As a result, the linearization to $e^x \cos x$ at 0 is

$$L(x) = x + 1.$$

Now find the values of x for which the linear approximation is accurate to within 0.1.

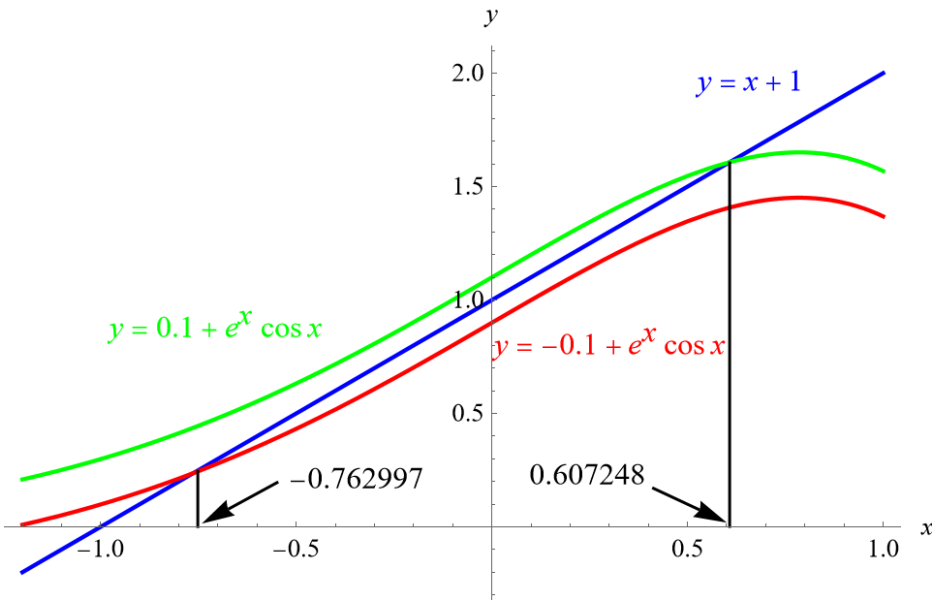
$$|e^x \cos x - (x + 1)| < 0.1$$

$$|(x + 1) - e^x \cos x| < 0.1$$

$$-0.1 < (x + 1) - e^x \cos x < 0.1$$

$$-0.1 + e^x \cos x < x + 1 < 0.1 + e^x \cos x$$

Plot each of these functions versus x.



The linear approximation stays between the curves for

$$-0.762997 < x < 0.607248.$$

This is the interval that the linear approximation is accurate to within 0.1.